Supporting Students Who Struggle with Maths: Algebra

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Introduction

The Cambridge English Dictionary defines algebra as, “a part of mathematics in which signs and letters represent numbers.” Yet, algebra is much more than this. It provides the tools to represent problems and find solutions and is a major part of the secondary school mathematics curriculum. Despite its significance, many students find algebra abstract and challenging, perceiving it as uninteresting and irrelevant to everyday life. This negative mindset impacts students’ willingness and motivation to learn not just algebra, but all mathematics. This article explores why students struggle with algebra and how to help them overcome these challenges, unlocking the door to problem-solving and a life-long joy for mathematics.

1. Understanding and Supporting Number Sense and Dyscalculia in Schools

Number sense is an intuitive grasp of numbers and their relationships. Students with strong number sense will understand the equation $\frac{x}{2} = 2.5$ in the following ways:

- Half of x is 2.5.
- Finding half of a number involves dividing it by 2.
- Dividing a number by 2 means splitting it into two equal parts.
- Doubling is the inverse of halving.
- To double a number, multiply it by 2.
- Since 2.5 is half of 5, x is 5.
- Double 2.5 is 5, so the solution is correct.

Students with a poor sense of numbers may find $\frac{x}{2}$ = 2.5 meaningless. They may also harbour misconceptions about numbers that impede problem-solving. In my experience supporting students in mathematics, some believe odd numbers cannot be halved or the answer to an algebraic equation cannot be a decimal number. Had the equation been presented as 2.5 = $\frac{x}{2}$, some students would have been unable to solve it because x was “on the wrong side”. These misconceptions and a lack of sense of numbers and how they relate will hinder students’ ability to be successful in algebra. They may also be indicative of an underlying learning difficulty.

Dyscalculia is “a specific and persistent difficulty in understanding numbers which can lead to a diverse range of difficulties with mathematics*” (*SASC, The Specific Learning Difficulties Assessment Standards Committee, 2019). An estimated 5 to 8% of school-age children have dyscalculia (Morsanyi et al., 2018), which means that in a mathematics class of 30 students, at least two learners may have severe difficulties that require specialist support and specific arrangements to access education effectively. In ability-based classrooms, this percentage may be even higher. Despite this prevalence, children with dyscalculia are 100 times less likely to be diagnosed compared to their peers with dyslexia. Even when diagnosed, there is no standard procedure to support these students adequately.

Identifying the signs of dyscalculia should be a core part of teacher training. Furthermore, if a teacher suspects a pupil has mathematical learning difficulties, they should know what steps to take to address the issue. University College London (UCL) has produced a toolkit to help teachers and parents identify dyscalculia, which can be accessed here. This toolkit identifies the signs

of dyscalculia and mathematical learning difficulties while providing information on how to get support for children with dyscalculia (see Figure 1).

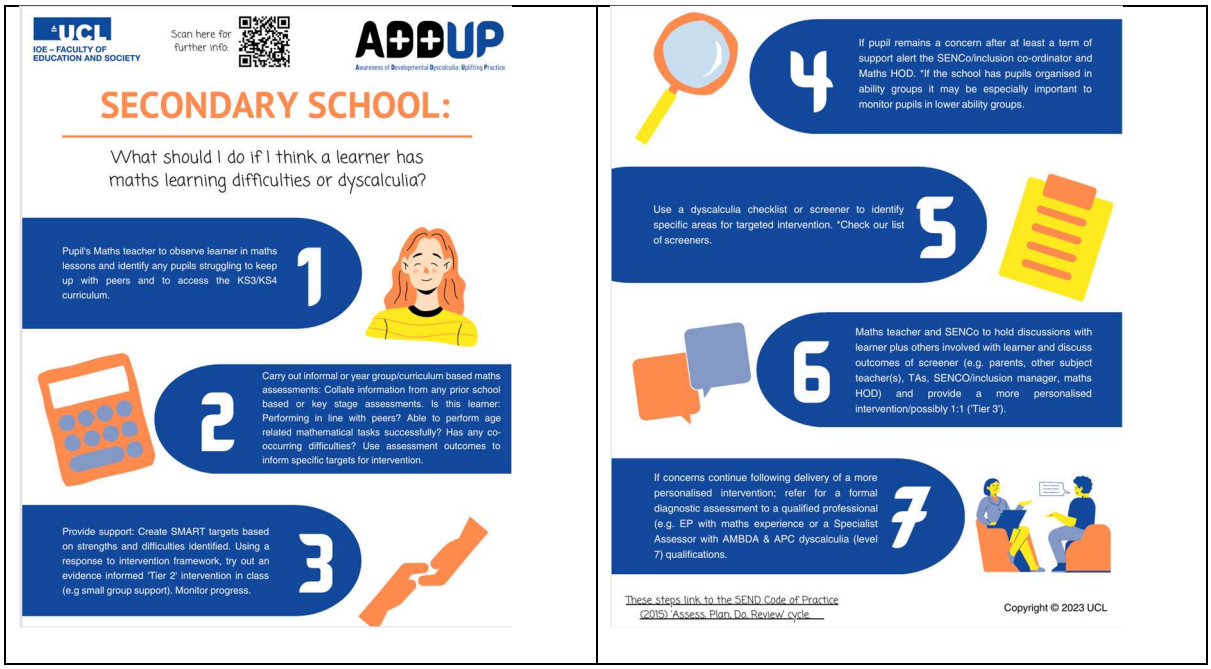

Figure 1. What should I do if I think a learner has maths learning difficulties or dyscalculia? (Van Herwegen, et al., 2022). Reproduced with permission.

Addressing the challenges faced by neurodiverse students or those with poor number sense and dyscalculia is crucial for their success in algebra. Yet traditional algebra teaching methods may be ineffective, and these students will require a different approach. For more information and ideas on how to teach algebra to neurodiverse pupils, see GCSE Maths for Neurodivergent Learners by Hornigold and Jewell (2022). The Dyscalculia Network is also an excellent source of ideas and information for educators.

2. The Importance of Basic Arithmetic Fluency in Algebra

To solve the equation $3 + 2x = 21$, a student must subtract 3 from 21, resulting in 18, and then divide 18 by 3 to find x = 9. Students proficient in basic arithmetic facts may solve such problems quickly and easily. For example, they might

immediately recognise that 21 is in the three times tables, and 3 less than 21 is 18, which can be divided by 2.

By contrast, a lack of fluency in basic arithmetic facts can significantly hinder progress in algebra. Students unable to recall these facts are often trapped in developmentally immature strategies such as counting in ones on their fingers (e.g., 21 take 1 is 20, 20 take 1 is 19, 19 take 1 is 18) and step counting through multiplication tables (e.g., 1 x 3 = 3, 2 x 3 = 6, … , 9 x 3 = 27). These methods are time-consuming and error-prone, leading to frustration, anxiety, reduced confidence and a dislike of algebra.

For students to succeed in algebra, they must have a firm foundation in addition, subtraction, multiplication, and division. This does not mean rote memorisation of arithmetic facts without understanding. Rather students should be taught for deep conceptual understanding and shown strategies for deriving basic arithmetic facts. For instance:

- **Addition and subtraction**: Use doubles and near-doubles strategies.
- **Multiplication**: Focus on the 1-, 2-, 5-, and 10-times tables, using these as anchors to reason about other facts.
- **Division**: Explicitly teach division as the inverse of multiplication so that the same core facts act as anchors.

By fostering logical reasoning and reducing the number of facts that must be memorised, students can develop confidence and efficiency in algebra. Chinn's excellent series of books – "What to do when you can't… (2009)" - are full of information and ideas on how to teach basic arithmetic fluency to neurodiverse pupils, as is his website Maths Explained**.**

3. The Role of Working Memory in Algebra

Working memory is the cognitive system where information is held and manipulated simultaneously. Individuals have a unique working memory capacity, which can be particularly impacted in those with SpLDs or SpLD traits. Working memory capacity cannot be increased. However, educators can support students to use their working memories more effectively.

Working memory plays a crucial role in algebra. Consider the equation $\frac{m-12}{3} = -3$. To solve this, a student must:

- Hold the equation in working memory while simultaneously retrieving the solving procedure from long-term memory.
- Multiply both sides of the equation by 3, retrieving the rules for multiplying negative and positive numbers, resulting in $m - 12 = -9$.
- Hold the partial product (−9) in memory while adding 12 ($m = 3$).
- Perform these steps without a calculator.
- Verify the solution by substituting 3 back into the equation.

Students proficient in basic arithmetic facts and familiar with solving two-step algebraic equations use their long-term memory to free up valuable working memory space. Conversely, students relying on developmentally immature counting strategies with poor memory for mathematical procedures may quickly overwhelm their working memory. This can be seen when students lose their place in problems, forget procedural steps, or fail to recall necessary information.

Mathematical educators can help students manage their working memory more effectively and improve their algebraic abilities. Gathercole and Alloway (2007)

have produced an excellent guide on supporting working memory in the classroom. Key insights include:

- **Chunk information**: Break information into smaller "chunks" to make it easier to manage.
- **Repeat important information**: Repeat important information and encourage students to rehearse instructions or steps.
- **Encourage external memory aids**: Allow students to use calculators, multiplication squares and/or number lines if teaching problem-solving rather than testing recall of mathematical facts.
- **Scaffold tasks**: Tasks should be highly structured initially. This can be reduced as students grow in confidence and understanding.
- **Monitor students**: Check in with students. Are they on track? Do they understand what to do?
- **Practice and overlearning**: Provide repeated opportunities to practice key skills and concepts until they become automatic, bypassing the student's working memory and relying instead on their long-term memory.

4. Supporting the Transfer of Information to Long-Term Memory in Students with SpLDs or with SpLD Traits

Information in working memory will be transferred into long-term memory for storage and subsequent reuse. Difficulty transferring information into long-term memory is a key characteristic of individuals with SpLDs such as dyscalculia and dyslexia. Many learners, including those not formally diagnosed with SpLDs, may experience similar difficulties. Learning how to solve algebraic equations will be

especially challenging for these learners as the rules may seem abstract, meaningless and hard to remember.

There are several ways to help these students, including:

- **Linking new information to previously stored information.** A student may be able to solve the missing number problem 10 - □ = 5 but fail to understand what to do with the equation 10 – x = 5. Linking the two, explicitly and repeatedly, students can form the connection and use the same step – inverse operations - to solve the missing number problem to solve the algebraic equation.
- **Making the information multisensory.** Information can be stored in working memory in a visuospatial or an auditory format. Students may have one memory channel that is weaker than the other. Multisensory teaching allows these students to use their stronger memory channel to overcome their weaker one, increasing the chance that information will be transferred to their long-term store. Clothesline Math by Shore (2018) is an excellent resource for how to make algebra teaching multisensory in a way that fosters logical reasoning and deepens understanding (see Figure 2).
- **Making the information meaningful.** Students often see algebra as irrelevant to their everyday life. They do not realise that algebraic equations underlie much of what they do and the decisions they make. Relating algebra to something students do or are interested in (see Box 1) will make it easier to understand and remember.

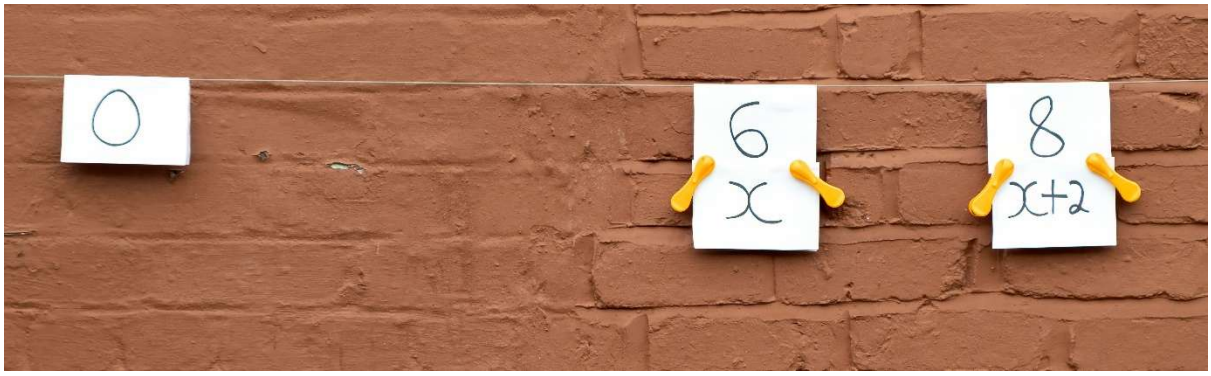

Figure 2. Solving an equation on a washing line.

Real Life Example

You want to buy a concert ticket that costs £100. You have £40 in savings. How much more do you save each week for the concert in three months? Let:

- x = the number of months you have to save;
- 40 be how much you already have saved,
- and 100 be the total cost of the ticket.

$$\frac{x}{3} + 40 = 100$$

Box 1. Real-life example of an algebra problem.

5. Supporting Students' Transition from Concrete to Abstract Thinking in Algebra

As students journey through school, they move from concrete, hands-on learning to an abstract, more theoretical understanding of concepts. Students need support to move from concrete to abstract thinking. Yet not only is algebra an abstract concept, but it is also almost always taught abstractly. This can make it hard for students to learn.

Take the problem: simplify the expression $x^2 + x + 2y - y$. As educators, how many times have you seen the solution: $x^3 + y$? How many times have you corrected this misunderstanding to witness the same student repeat it? I believe that this mistake – not knowing what like terms are - is because the student has not spent sufficient time learning algebra at the concrete level. Consequently, they do not have a strong internal representation of like terms and how they can be collected.

To overcome this problem, algebra should be taught concretely using manipulatives such as algebra tiles and balance scales. Virtual manipulatives are also freely available and are an excellent resource, especially for older students. Once students are secure at the concrete level, they can transition to more visual representations of algebraic problems, including drawings of scales or reasoning on number lines. Finally, students can transition to an abstract representation of algebra when ready. Figure 4 is an example of how I used this approach when tutoring a student with dyslexia to solve two-step equations.

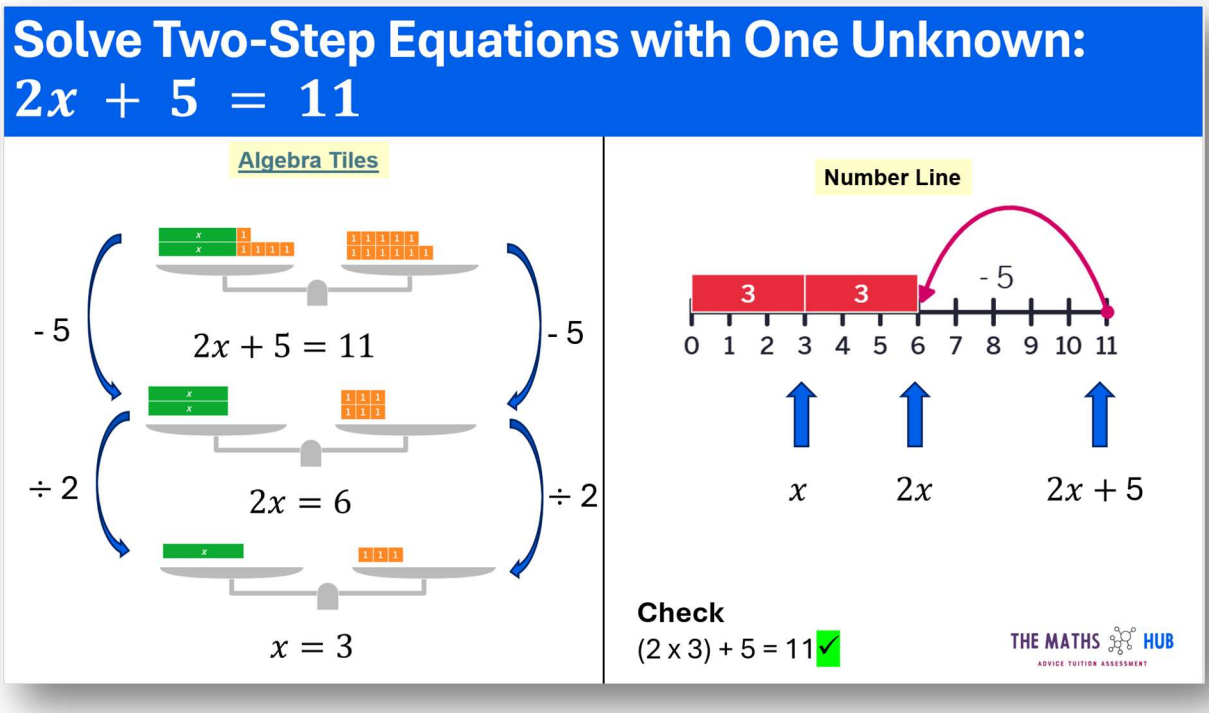

Figure 4. Using balance scales and number lines to solve two-step equations. Steps were animated and revealed in succession. Images created using free virtual manipulatives available at Polypad.com.

6. The Importance of Algebraic Vocabulary in Mathematics Education

Mathematics is a discipline with a unique vocabulary and algebra is no exception. Students who do not grasp basic algebra vocabulary will struggle to understand instructions, communicate their knowledge, and solve problems. In contrast, a solid understanding of algebra vocabulary can greatly enhance students' ability to deconstruct and solve algebra problems more effectively.

To ensure comprehension, algebraic vocabulary should be explicitly taught at the start of the topic. Teaching vocabulary does not guarantee understanding or retention, however. Therefore, regularly revisiting vocabulary is essential. Employing low-stakes, games-based approaches such as hangman (see Figure 5), bingo or card-sorting exercises can make this process engaging and effective.

7. Helping Students Understand Algebraic Conventions

A convention is a rule that people have agreed to follow. Algebraic conventions are specific rules mathematicians have established to facilitate problem-solving. These rules include omitting multiplication signs; writing the number first in a product; arranging letters in alphabetic order in terms; and following BIDMAS (Brackets, Indices, Division and Multiplication, Addition and Subtraction) order of operations. Further conventions dictate how to handle fractions, negative numbers and brackets.

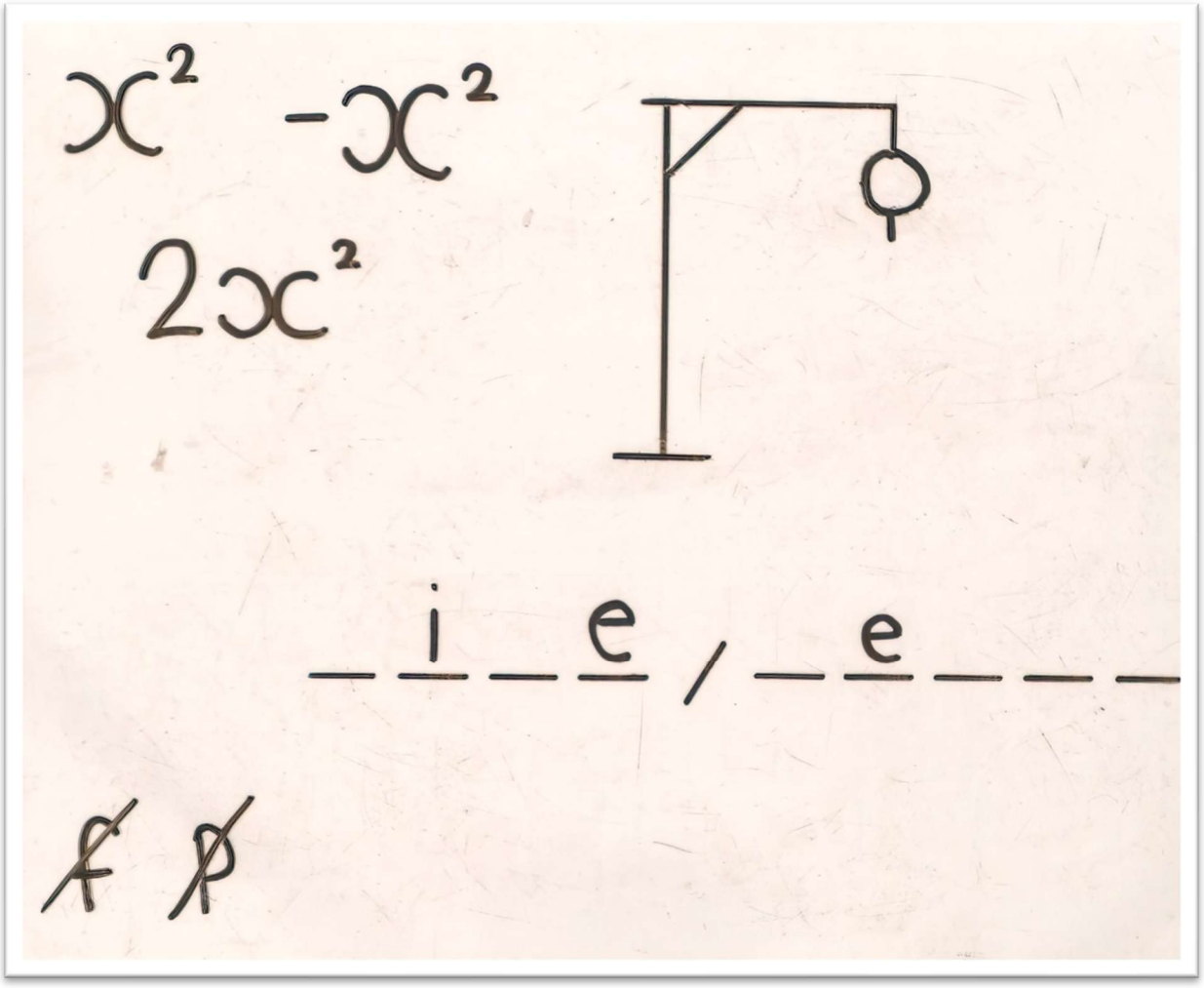

Figure 5. Using hangman to reinforce recall of algebraic vocabulary.

However, students find these conventions arbitrary and difficult to remember. Therefore, just like algebraic vocabulary, these conventions should be explicitly taught and reinforced regularly. Using engaging and interactive teaching methods can help students internalise these rules more effectively.

8. Make Word Problems an Integral Part of Algebra Lessons

In algebra, students are often expected to solve word problems. However, some students do not understand what they have been asked to do or how to approach the problem. This difficulty may be particularly pronounced in those with underlying literacy challenges.

The following suggestions may support these students:

- **Read the problem several times.**
- **Highlight important information.**
- **Sketch the problem out.**
- **Represent unknowns with letters.**
- **Break the problem down into manageable chunks.**
- **Put the chunks back together.**

See Figure 6 for an example of how I have supported a pupil in solving a word problem using this approach.

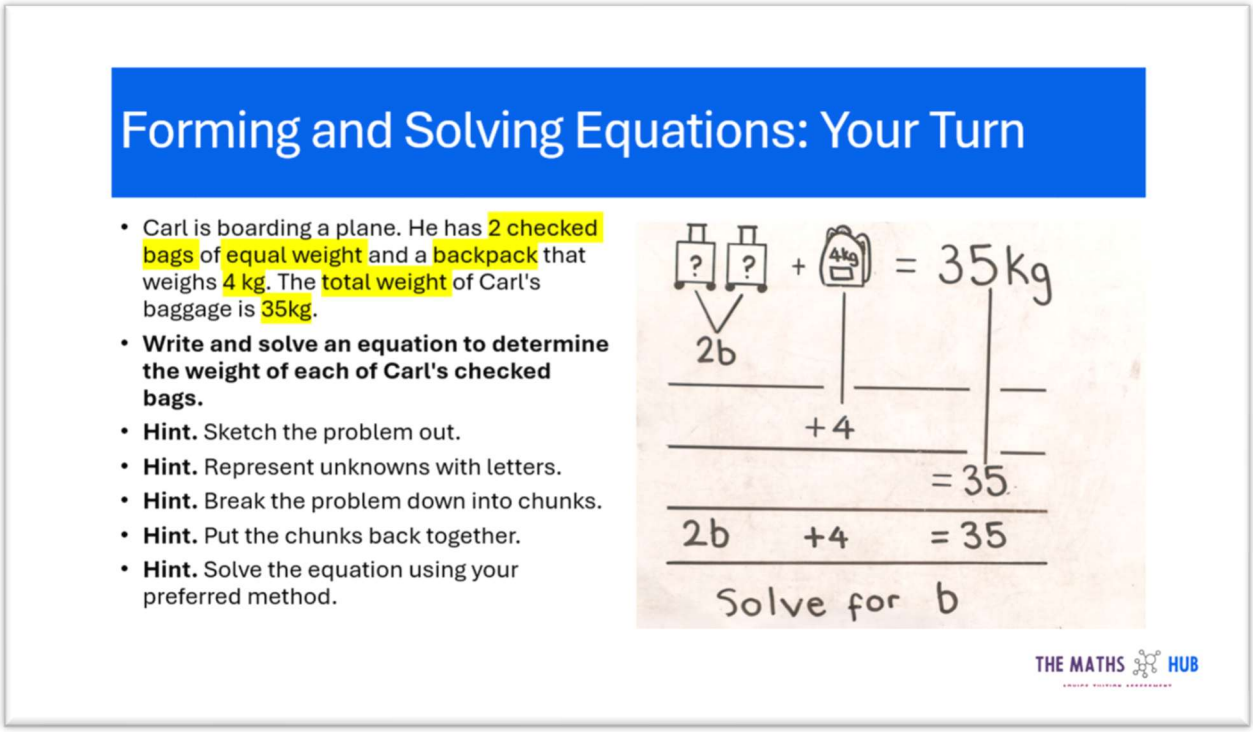

Figure 6. Solving algebra word problems.

It is essential to encourage students to start the problem-solving process, even if they do not reach the final answer. This “can-do attitude” fosters resilience and may gain marks in an exam that would make the difference between passing and failing.

Conclusion

Algebra represents a significant part of the secondary school curriculum. However, many students perceive it as abstract, challenging, and irrelevant to everyday life. This negative mindset undermines their willingness and motivation to learn, not just in algebra, but in all areas of mathematics. As educators, we must work to change this mindset. We must transform algebra from a subject students dread to a challenge they eagerly engage with. We can achieve this by building strong mathematical foundations in our students; supporting their learning needs; and making manipulatives a must-have in every classroom. In doing so we will help students develop critical thinking, abstract reasoning and other problem-solving skills, preparing them to become employees of the future.

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