

Understanding Division

The Language of Division

Divisor

The number we divide by in a division calculation.

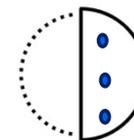
Quotient

The answer you get when you divide one number by another.

Dividend
The number you want to divide up in a division calculation.

$$56 \div 7 = 8$$

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$



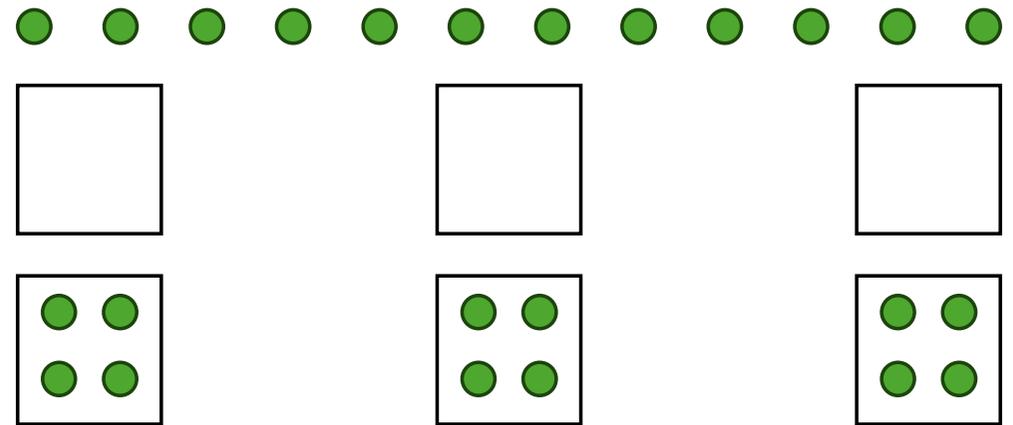
Models of Division: Sharing and Grouping

There are two models of division: sharing and grouping. The sharing model involves distributing an amount fairly. The grouping model is the opposite of multiplication facts. It consists of putting an amount into equal groups.

The sharing model of division:

- Take 12 counters.
- Share them equally among three containers.
- How many counters are in each container?
- This is the final answer.

Example: $12 \div 3 =$



There are four counters in each container.

$$12 \div 3 = 4$$

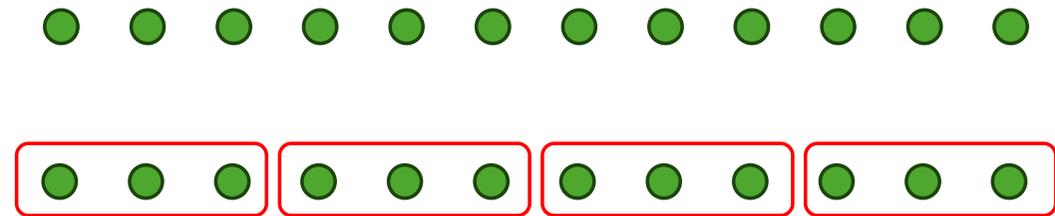
Models of Division: Sharing and Grouping

There are two models of division: sharing and grouping. The sharing model involves distributing an amount fairly. The grouping model is the opposite of multiplication facts. It consists of putting an amount into equal groups.

The grouping model of division:

- Take 12 counters.
- Arrange them into groups of three.
- How many groups of three are there?
- This is the final answer.

Example: $12 \div 3 =$



There are four groups of three

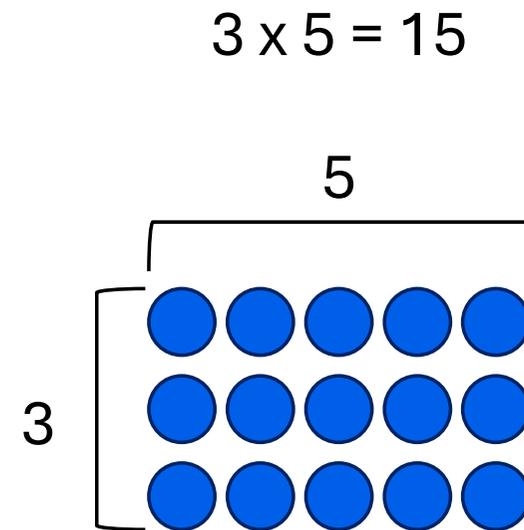
$$3 \times 4 = 12, \text{ so}$$
$$12 \div 3 = 4$$

The Area Model of Multiplication

The area model of multiplication is a visual model that represents multiplication as finding the area of a rectangle.

The area model of division:

- I arrange counters in 3 rows of 5 counters each.
- There are 15 counters in total.
- $3 \times 5 = 15$
- 3 (number of rows) \times 5 (counters in each row/columns) = 15 (area of array or product)



Using The Area Model of Multiplication to Divide

Division is the inverse of multiplication. That means we can use the area model to think about division calculations.

The area model of division:

- Division asks: “If I know the area of a rectangle and one side, what is the missing side?”
- I need 3 lots of something to make an area of 15.
- $3 \times 5 = 15$
- So, $15 \div 3 = 5$

$$15 \div 3 = ?$$

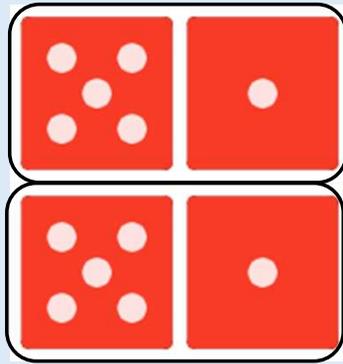
?



Doubling and Halving

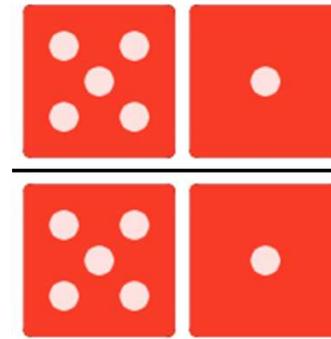
Doubling and halving are both special examples of multiplication and division. Doubling is multiplying a number by two, whereas halving is dividing a number by two.

Doubling (x 2)



$$6 \times 2 = 12$$

Halving ($\div 2$)



$$12 \div 2 = 6$$

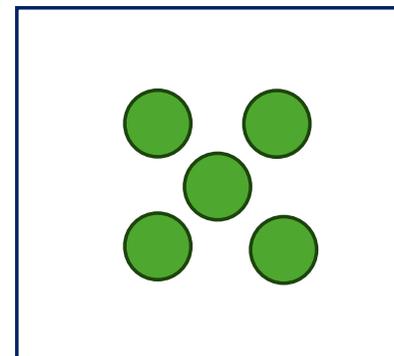
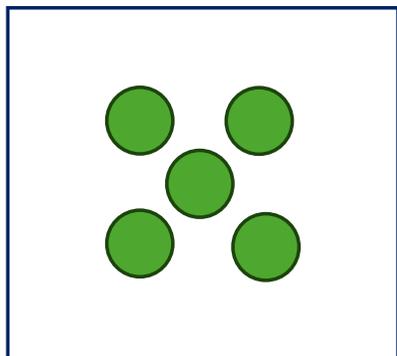
Remainders

Not all divisions work out exactly. Sometimes there is a remainder. Be careful when interpreting remainders in division problems because the context of the problem determines how the remainder should be handled.



Sharing may not be equal. Five in each container with one remainder.

$$11 \div 2 = 5 \text{ r}1$$



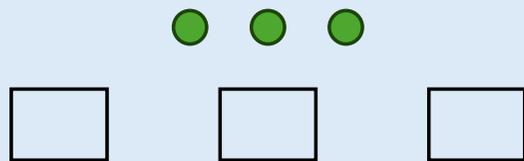
Dividing a Number By Itself, One and Zero

Dividing a number by itself, one and zero can be tricky. However, if you use counters and containers to model the problem, it becomes much easier to understand.

Dividing a Number By Itself

$$3 \div 3 =$$

I have three counters. I share them equally among three containers.



How many counters are in each container?



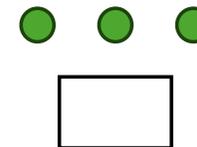
There is one counter in each container.

$$3 \div 3 = 1$$

Dividing a Number By One

$$3 \div 1 =$$

I have three counters. I share them equally among one container.



How many counters are in the container?



There are three counters in the container.

$$3 \div 1 = 3$$

Dividing a Number By Itself, One and Zero

Dividing a number by itself, one and zero can be tricky. However, if you use counters and containers to model the problem, it becomes much easier to understand.

Dividing Zero

$$0 \div 3 =$$

I have no counters. I have three containers.



How many counters are in each container?

Zero. I cannot share counters when there are none to share.

$$0 \div 3 = 0$$

Dividing By Zero

$$3 \div 0 =$$

I have three counters. I have no containers.



I cannot share counters when there is no container.

$$\cancel{3 \div 0 =}$$

Dividing a Number By Itself, One and Zero

Dividing a number by itself, one and zero can be tricky. However, if you use counters and containers to model the problem, it becomes much easier to understand.

Dividing a number by itself

- Any number (except zero) divided by itself equals **1**.
- Example: $8 \div 8 = 1$

Dividing a number by 1

- Any number divided by 1 remains **the same**.
- Example, $15 \div 1 = 15$

Dividing zero

- Zero divided by any number (except zero itself) is always 0.
- Example: $0 \div 5 = 0$

Dividing a number by 0

- You **cannot** divide by zero.

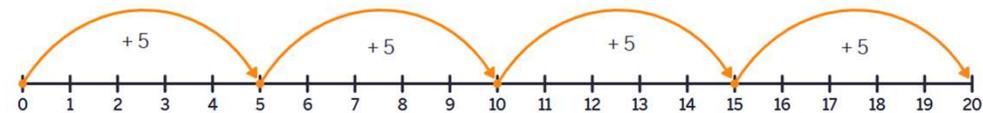
Using a Number Line for Division: Repeated Addition

Number lines break division problems into smaller, manageable steps, making them easier to visualise.

How to Use a Number Line to Divide:

- **Start at zero.** Begin at zero on the number line.
- **Add the divisor.** Jump forward by the number you are dividing by, drawing an arc representing each step. Label each arc.
- **Repeat addition.** Continue adding the divisor until you reach or you get as close as possible to the dividend. Any difference left is the remainder.
- **Count the jumps.** The number of times you added the divisor is the quotient – the final answer to the division problem.
- **State the final result.** Clearly state the final answer, including any remainder if applicable.

Example: $20 \div 5 =$



I added five four times to make 20 .

$$20 \div 5 = 4$$

Using a Multiplication Square to Divide (No Remainder)

Answers to division problems can be checked using a times-table square, if the answer falls within the 12×12 facts. The checking method changes slightly depending on whether the division has a remainder or not.

How to use a multiplication square to divide:

- **Find the divisor:** Locate the number you are dividing by in the top row.
- **Track down to the dividend :** Follow the column down until you reach the number you are dividing.
- **Track across:** Move across to the end of the row until you find how many times the divisor fits into the dividend.
- **State the final answer:** Clearly state the final answer.

Example: $27 \div 9 =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

There are three nines in 27.

$$27 \div 9 = 3$$

Using a Multiplication Square to Divide (Remainder)

Answers to division problems can be checked using a times-table square, if the answer falls within the 12×12 facts. The checking method changes slightly depending on whether the division has a remainder or not.

How to use a multiplication square to divide:

- **Find the divisor:** Locate the number you are dividing by in the top row.
- **Track down to the dividend :** Follow the column down until reach the number you are dividing or the closest match.
- **Track across :** Move across to the end of the row until you find how many times the divisor fits into the dividend.
- **Calculate the remainder:** Subtract the result from the dividend to find any remainder.
- **State the final answer:** Present the quotient and remainder clearly.

Example: $29 \div 9 =$

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

There are three nines in 29.

$$29 - 27 = 2$$

$$29 \div 9 = 3 \text{ remainder } 2$$

Visual Models of Division (No Remainder)

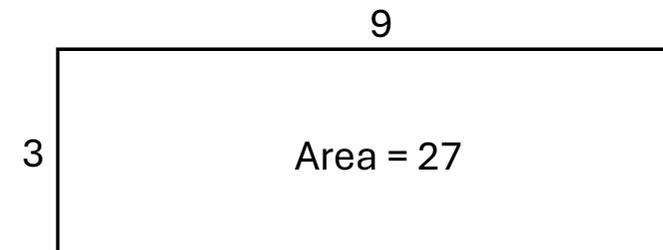
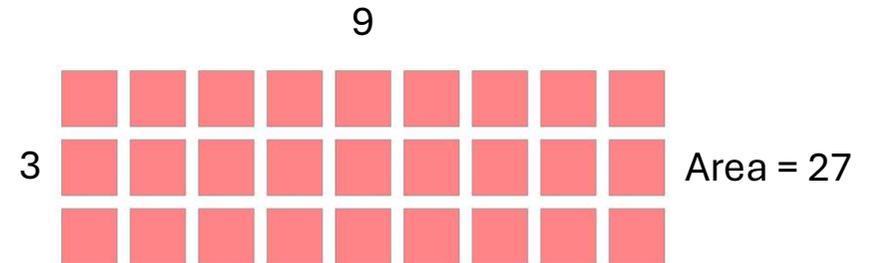
Visual models help break division into smaller, manageable steps.

They show how a dividend can be shared equally using simple, clear representations such as the area model and a number line. Answers can be checked on a times table square.

How to divide using visual models:

- **Reframe the question:** Ask, “If I share 27 equally among 3 rows, how long will each row be?”
- **Area model:** Place counters or blocks in rows that match the divisor. Continue placing them side-by-side until all counters are used. Label the sides and area.
- **Sketch:** Outline the counters or blocks and label the sides and the area.

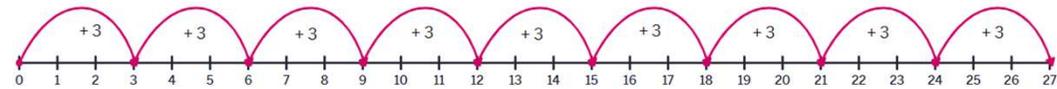
Example: $27 \div 3 =$



Visual Models of Division (No Remainder)

- **Number line:** Represent the calculation using repeated addition or subtraction of the **divisor** on a number line.
- **Verify:** Use a times table square to confirm the result.
- **State the final answer:** Clearly state your final answer.

Example: $27 \div 3 =$



	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

There are nine threes in 27.

$$9 \times 3 = 27$$

$$27 \div 3 = 9$$

Visual Models of Division (With a Remainder)

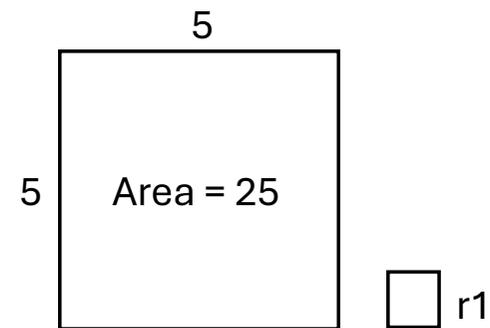
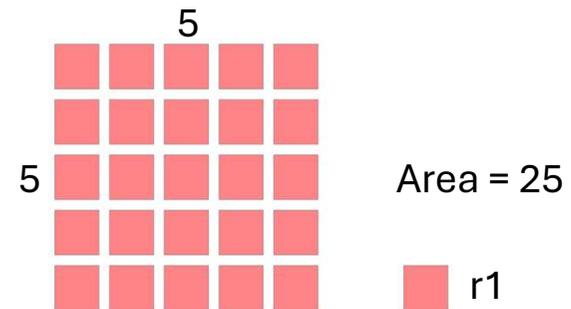
Visual models help break division into smaller, manageable steps.

They show how a dividend can be shared equally using simple, clear representations. However, not all divisions work out evenly, so some calculations have a remainder.

How to divide using visual models:

- **Reframe the question:** Ask, “If I share 26 equally among 5 rows, how long will each row be?”
- **Area model:** Place counters or blocks in rows that match the divisor. Continue placing them side-by-side until all counters are used or no full row can be completed. Label the sides, the area and any leftover counters as the remainder.
- **Sketch:** Outline the counters or blocks and label the sides, the area and the remainder.

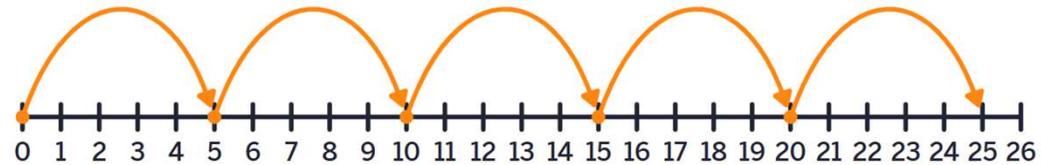
Example: $26 \div 5 =$



Visual Models of Division (With a Remainder)

- **Number line:** Represent the calculation using repeated addition or subtraction of the **divisor** on a number line. Stop when the next jump would go past the dividend. Record the remainder.
- **Verify:** Use a times table square to find the closest multiplication fact without going over the dividend.
- **State the final answer:** Clearly state your final answer and any remainder.

Example: $26 \div 5 =$



	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

There are five fives in 26 and a remainder of 1.

$$5 \times 5 = 25$$

$$26 \div 5 = 5 \text{ r}1$$

Acknowledgements

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