

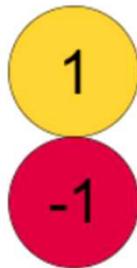
Operations with Negative and Positive Numbers

Add, Subtract, Multiply, Divide

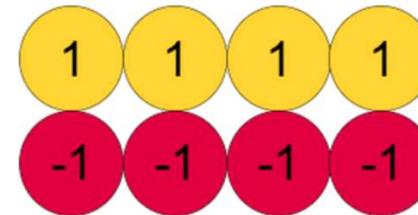
A Zero Pair

Negative numbers are less than zero and written with a sign, e.g. (-1). Positive numbers are greater than zero and usually do not have a sign, e.g. 1. You can represent (-1) and 1 using red and yellow counters. Together they make a zero pair. There are lots of ways to make zeros. Zero pairs are useful when thinking about operations with positive and negative numbers.

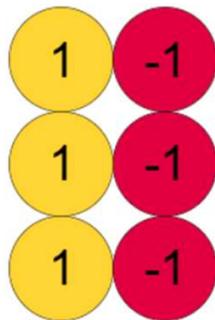
A Zero Pair



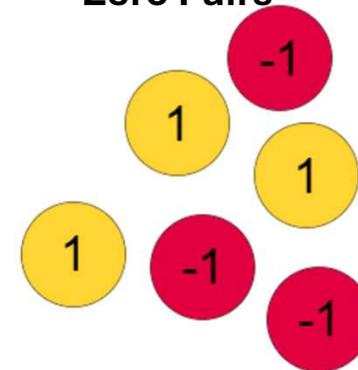
Zero Pairs



Zero Pairs

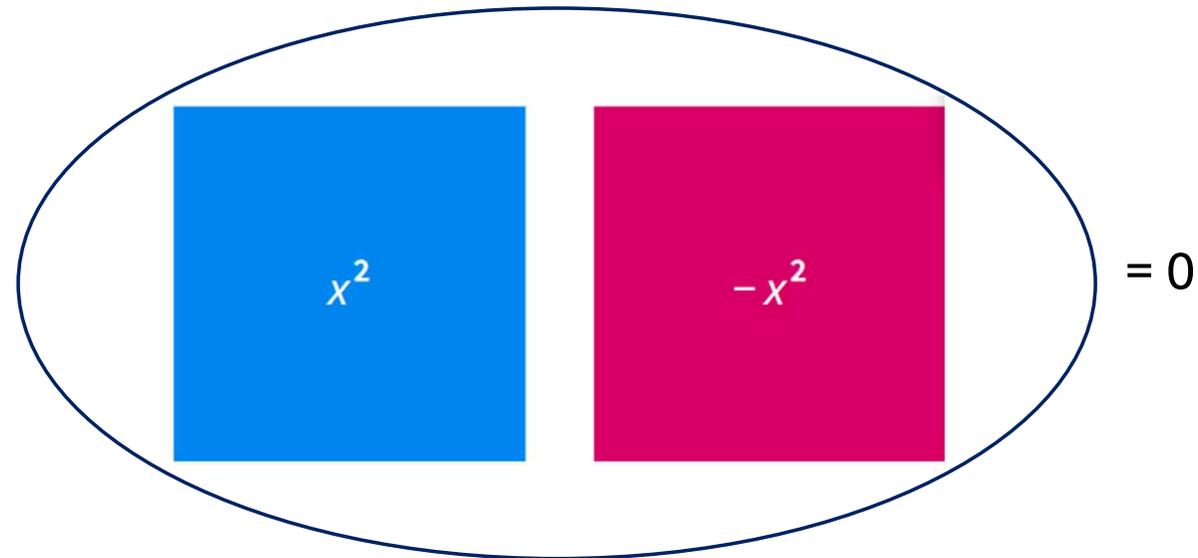
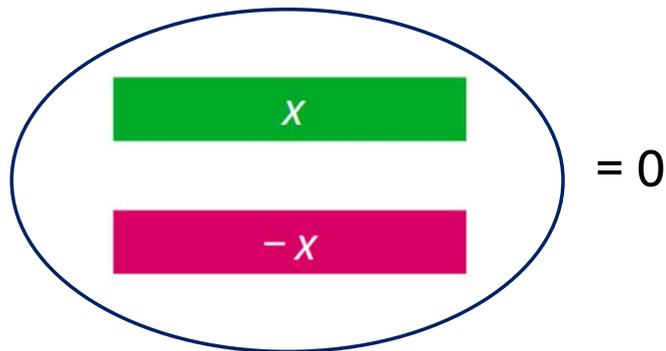


Zero Pairs



Other Zero Pairs

To model zero with algebra tiles means showing that equal and opposite tiles cancel each other out.

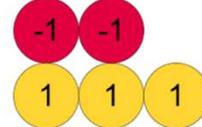


Adding and Subtracting Negative and Positive Numbers

You can show adding and subtracting positives and negatives with counters: yellow for positive, red for negative. Adding means putting counters together, subtracting means taking them away. If you don't have enough to subtract, make zero pairs, since one yellow and one red cancel out.

Adding a positive number

$2 + 3$  **Answer: 5**

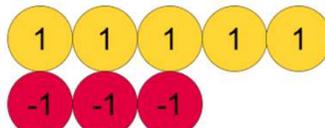
$(-2) + 3$  **Answer: 1**

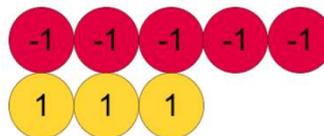
Subtracting a positive number

$6 - 3$  **Answer: 3**

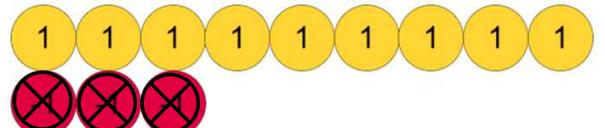
$(-6) - 3$  **Answer: (-9)**

Adding a negative number

$5 + (-3)$  **Answer: 2**

$(-5) + 3$  **Answer: (-2)**

Subtracting a negative number

$6 - (-3)$  **Answer: 9**

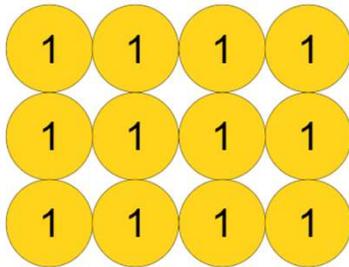
$(-6) - (-3)$  **Answer: (-3)**

Multiplying Negative and Positive Numbers

You can show multiplying positives and negatives with counters: yellow for positive, red for negative. Read the multiplication as repeated addition (or subtraction) of the same number and build an array to represent the calculation. If you don't have enough counters to take away, create zero pairs.

$$3 \times 4$$

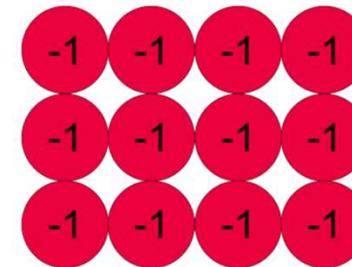
Add three lots of 4



Answer 12

$$3 \times (-4)$$

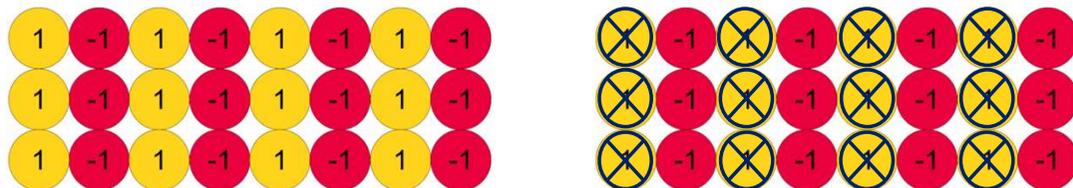
Add three lots of (-4)



Answer (-12)

$$(-3) \times 4$$

Takeaway three lots of 4

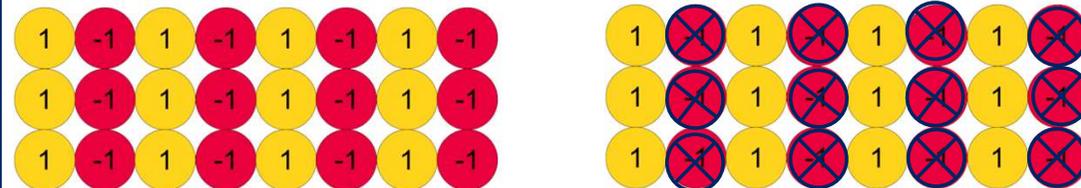


Create zero number pairs

Answer (-12)

$$(-3) \times (-4)$$

Takeaway three lots of (-4)



Create zero number pairs

Answer 12

Dividing Negative and Positive Numbers

Division is the inverse of multiplication. One way to understand division of negative and positive numbers is to ask, what do I need to repeatedly add (or take away) to get the number being divided?

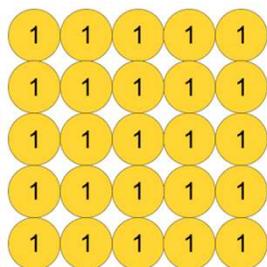
We can use red and yellow counters to model this.

$$25 \div 5 =$$

Add five lots of something to make 25

$$5 \times 5 = 25$$

$$\text{So, } 25 \div 5 = 5$$

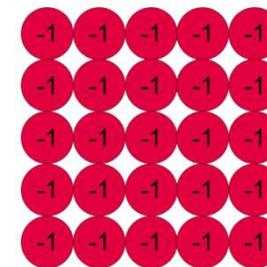


$$-25 \div 5 =$$

Add 5 lots of something to make -25

$$5 \times -5 = -25$$

$$\text{So, } -25 \div 5 = -5$$



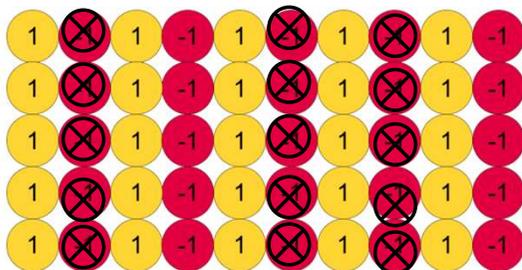
$$25 \div -5 =$$

Take away five lots of something to make 25

Create zero pairs

$$-5 \times -5 = 25$$

$$\text{So, } 25 \div -5 = -5$$



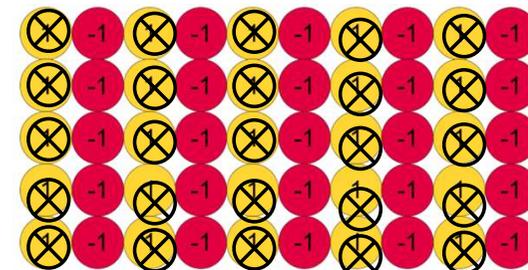
$$-25 \div -5 =$$

Take away five lots of something to make -25

Create zero pairs

$$-5 \times 5 = -25$$

$$\text{So, } -25 \div -5 = 5$$



Multiplication Grid with Negative Numbers

A multiplication square can be extended to include negative numbers. Rely on the patterns and symmetry in the grid to make it easier to complete.

To draw a multiplication square with negative numbers:

- **Draw your usual grid.** Start in the bottom right corner of your page, draw your standard multiplication square.
- **Extend the number lines.** Continue the rows backwards and columns upwards to include negative numbers.
- **Fill using patterns, not rules.** Extend each row backwards by subtracting the row number. Extend each column upwards by subtracting the column number. Look for symmetry and keep the pattern going.
- **Colour code and label.** Shade the four quadrants to highlight the zones of multiplication. Label all columns and rows clearly.
- **Ready to use.** Your square is now ready to multiply (and divide) negative and positive numbers.

x	-4	-3	-2	-1	0	1	2	3	4
-4	16	12	8	4	0	-4	-8	-12	-16
-3	12	9	6	3	0	-3	-6	-9	-12
-2	8	6	4	2	0	-2	-4	-6	-8
-1	4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0	0
1	-4	-3	-2	-1	0	1	2	3	4
2	-8	-6	-4	-2	0	2	4	6	8
3	-12	-9	-6	-3	0	3	6	9	12
4	-16	-12	-8	-4	0	4	8	12	16

Multiplication Grid with Negative Numbers – Pattern Method

A multiplication square can be extended to include negative numbers. Rely on the patterns and symmetry in the grid to make it easier to complete.

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

				-4				
				-3				
				-2				
				-1				
				0				
-4	-3	-2	-1	0	1	2	3	4
-8	-6	-4	-2	0	2	4	6	8
-12	-9	-6	-3	0	3	6	9	12
-16	-12	-8	-4	0	4	8	12	16

Multiplication Grid with Negative Numbers – Pattern Method

A multiplication square can be extended to include negative numbers. Rely on the patterns and symmetry in the grid to make it easier to complete.

					-4	-8	-12	-16
					-3	-6	-9	-12
					-2	-4	-6	-8
					-1	-2	-3	-4
					0	0	0	0
-4	-3	-2	-1	0	1	2	3	4
-8	-6	-4	-2	0	2	4	6	8
-12	-9	-6	-3	0	3	6	9	12
-16	-12	-8	-4	0	4	8	12	16

16	12	8	4	0	-4	-8	-12	-16
12	9	6	3	0	-3	-6	-9	-12
8	6	4	2	0	-2	-4	-6	-8
4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0
-4	-3	-2	-1	0	1	2	3	4
-8	-6	-4	-2	0	2	4	6	8
-12	-9	-6	-3	0	3	6	9	12
-16	-12	-8	-4	0	4	8	12	16

Multiplication Grid with Negative Numbers – Pattern Method

A multiplication square can be extended to include negative numbers. Rely on the patterns and symmetry in the grid to make it easier to complete.

x	-4	-3	-2	-1	0	1	2	3	4
-4	16	12	8	4	0	-4	-8	-12	-16
-3	12	9	6	3	0	-3	-6	-9	-12
-2	8	6	4	2	0	-2	-4	-6	-8
-1	4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0	0
1	-4	-3	-2	-1	0	1	2	3	4
2	-8	-6	-4	-2	0	2	4	6	8
3	-12	-9	-6	-3	0	3	6	9	12
4	-16	-12	-8	-4	0	4	8	12	16

x	-4	-3	-2	-1	0	1	2	3	4
-4	16	12	8	4	0	-4	-8	-12	-16
-3	12	9	6	3	0	-3	-6	-9	-12
-2	8	6	4	2	0	-2	-4	-6	-8
-1	4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0	0
1	-4	-3	-2	-1	0	1	2	3	4
2	-8	-6	-4	-2	0	2	4	6	8
3	-12	-9	-6	-3	0	3	6	9	12
4	-16	-12	-8	-4	0	4	8	12	16

Acknowledgements

- Images were created using free virtual manipulatives by Amplify available at [Polypad.com](https://www.polypad.com) and [Mathsbot.com](https://www.mathsbot.com)